Principles of Communications
ECS 332

Asst. Prof. Dr. Prapun Suksompong

prapun@siit.tu.ac.th

7. Pulse Modulation, ISI, and Pulse Shaping

Office Hours:

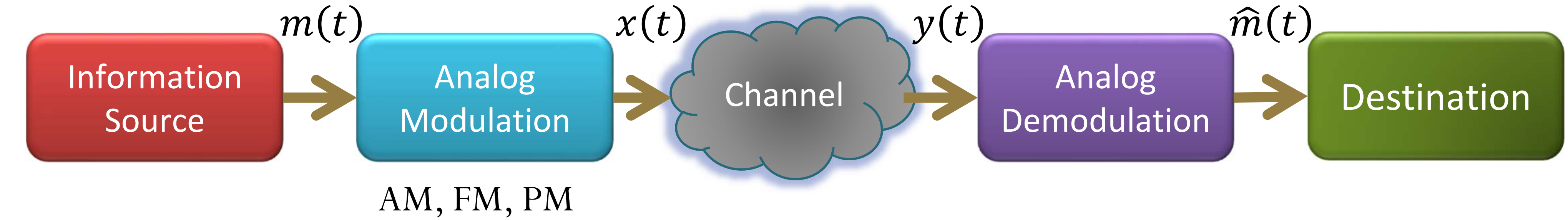
Check Google Calendar on the course website.

Dr.Prapun's Office:

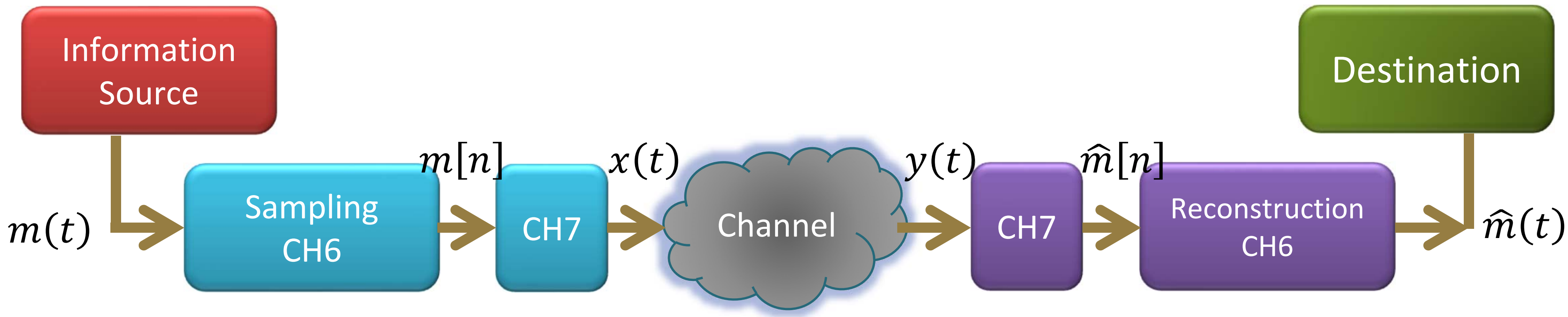
6th floor of Sirindhralai building, BKD

CH 3-5 vs. CH 6-7

- Chapters 3-5



- Chapters 6-7



ASCII in MATLAB

```
>> str='I love SIIT';
>> real(str)

ans =

    73    32   108   111   118   101    32    83    73    73    84

>> dec2base(str,2)

ans =

1001001
0100000
1101100
1101111
1110110
1100101
0100000
1010011
1001001
1001001
1010100
```

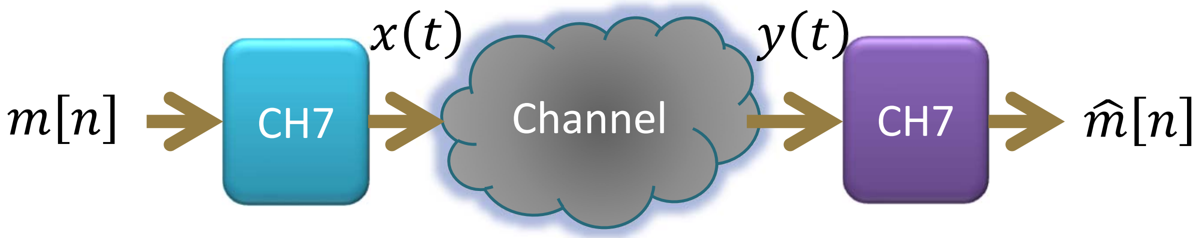
text string

(decimal) ASCII representation of the text string

binary (base 2) representation of the decimal numbers

Chapter 7

- **Start with a discrete-time signal** (a sequence of numbers $m[n]$).
- Goal: Convert $m[n]$ into an analog signal $x(t)$ that
  - has no "inter-symbol interference"
  - uses small bandwidth.



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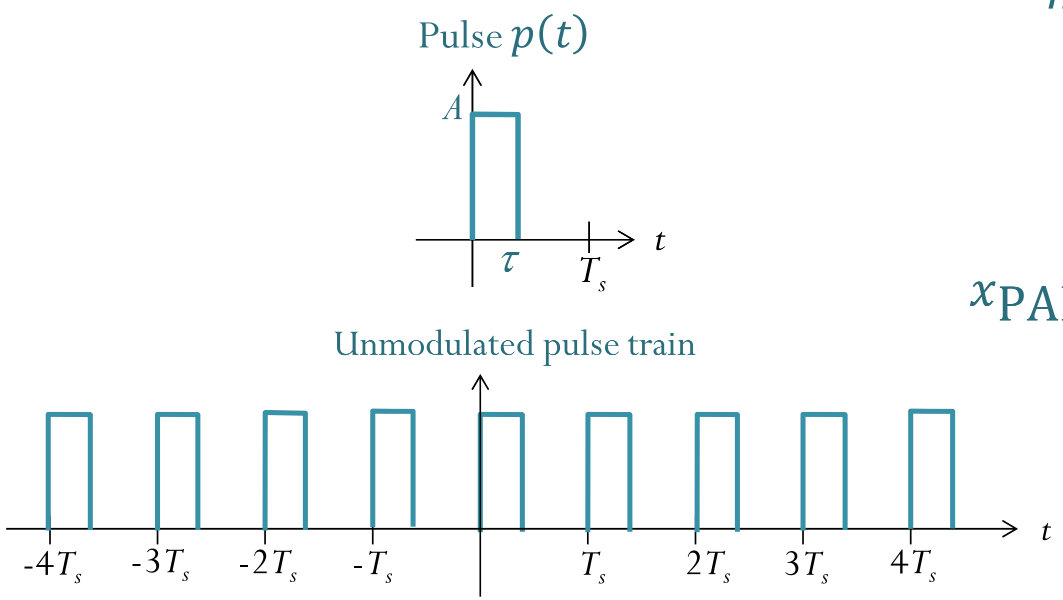
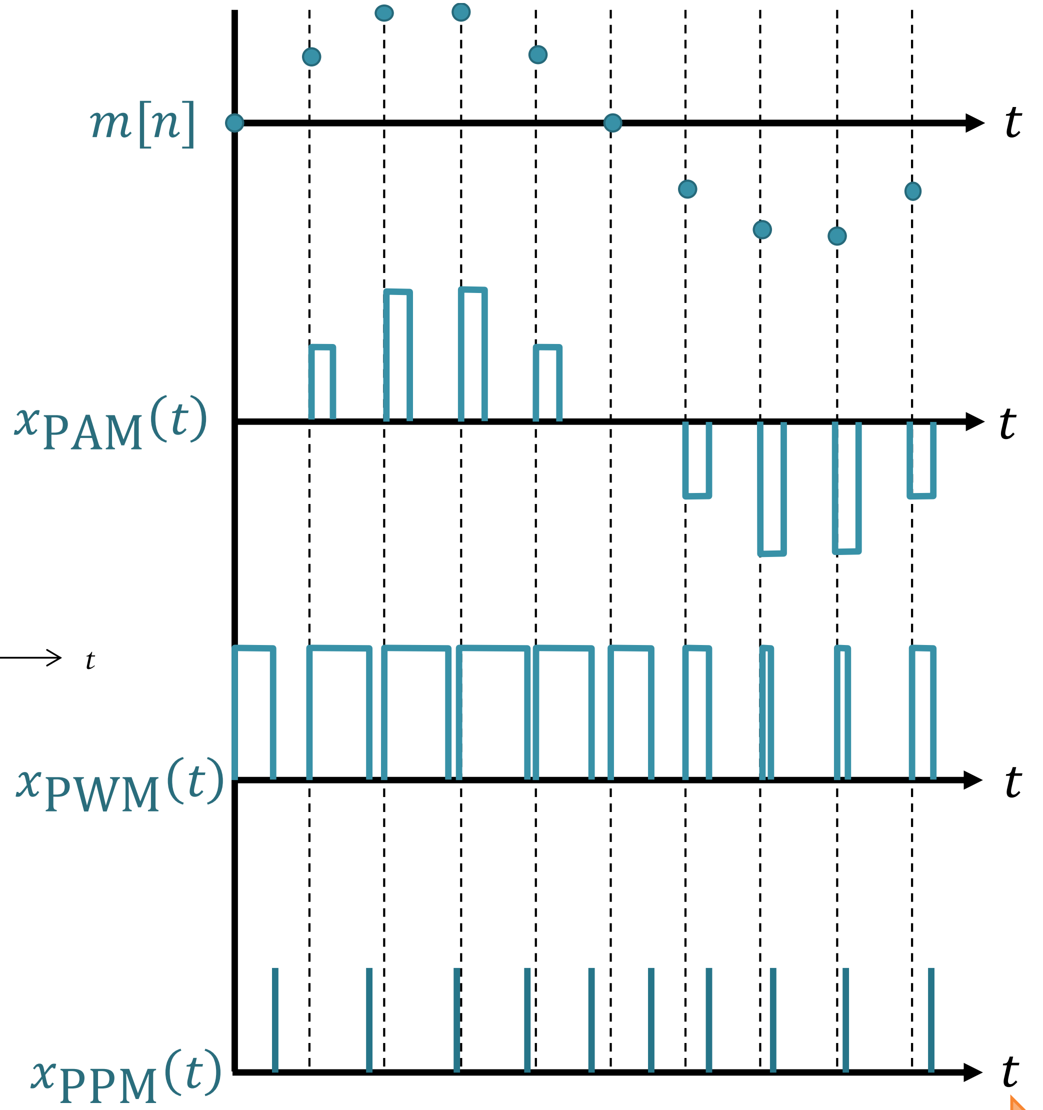
7.1 Analog Pulse Modulation

Analog Pulse Modulation





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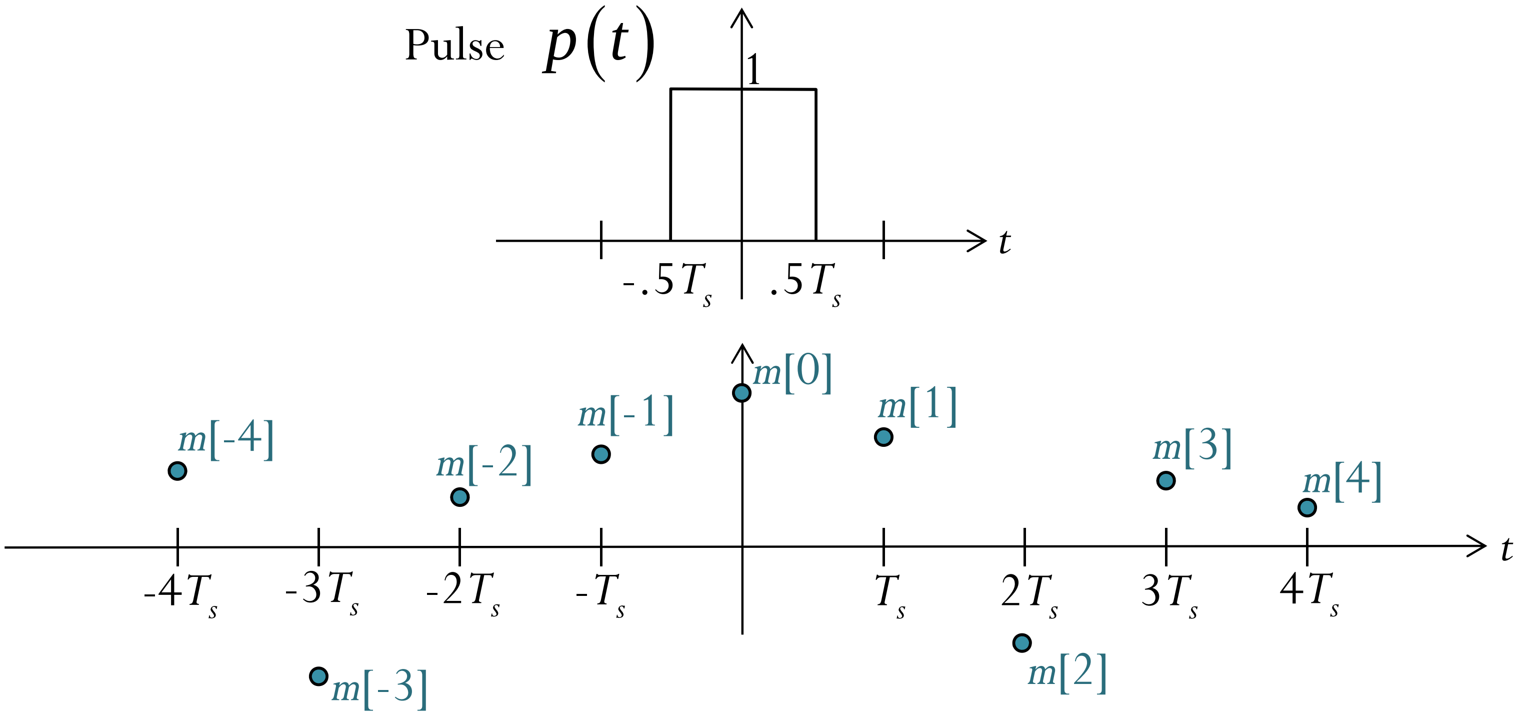
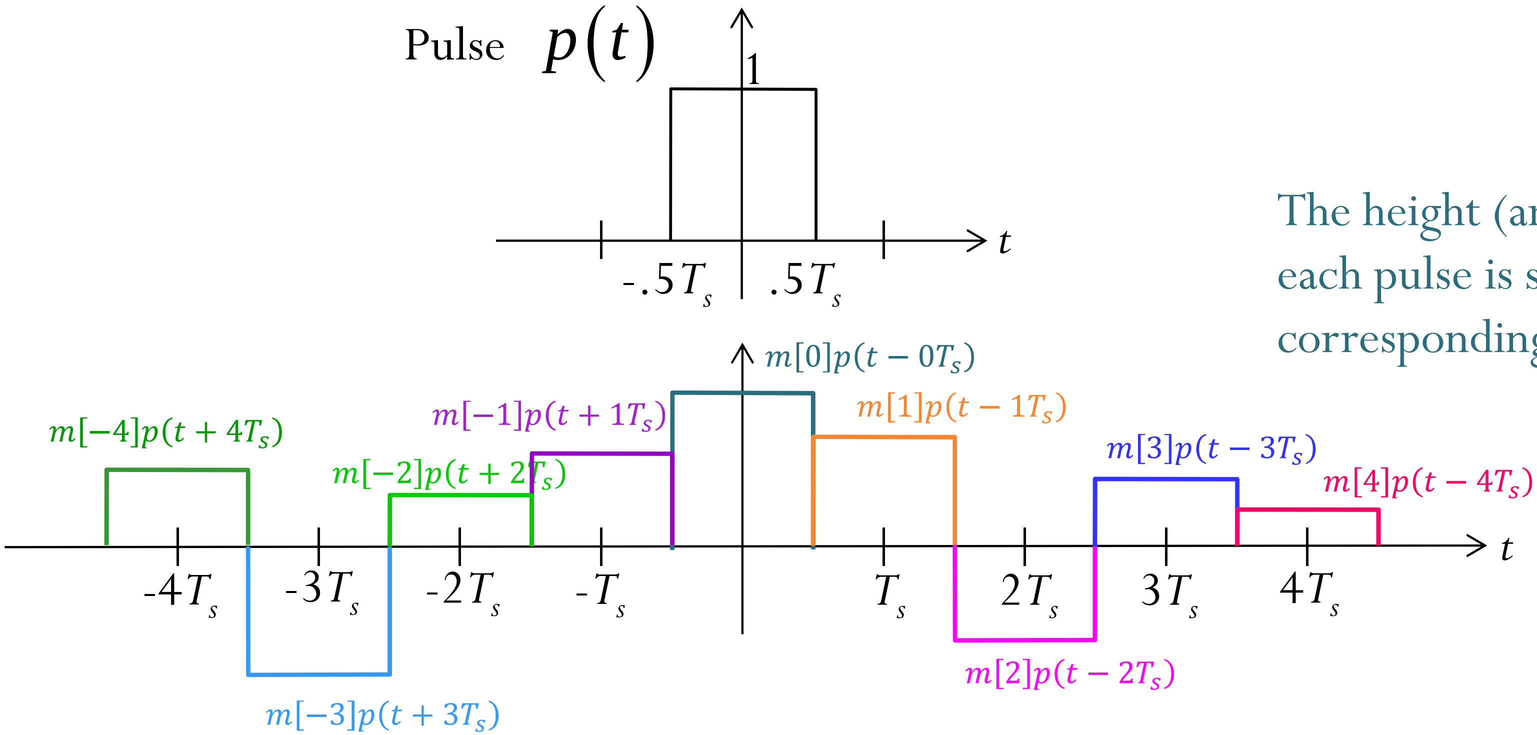
7.2 PAM

PAM: Example 7.11 (Figure 58)



$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n]\, p(t - nT_s)$$

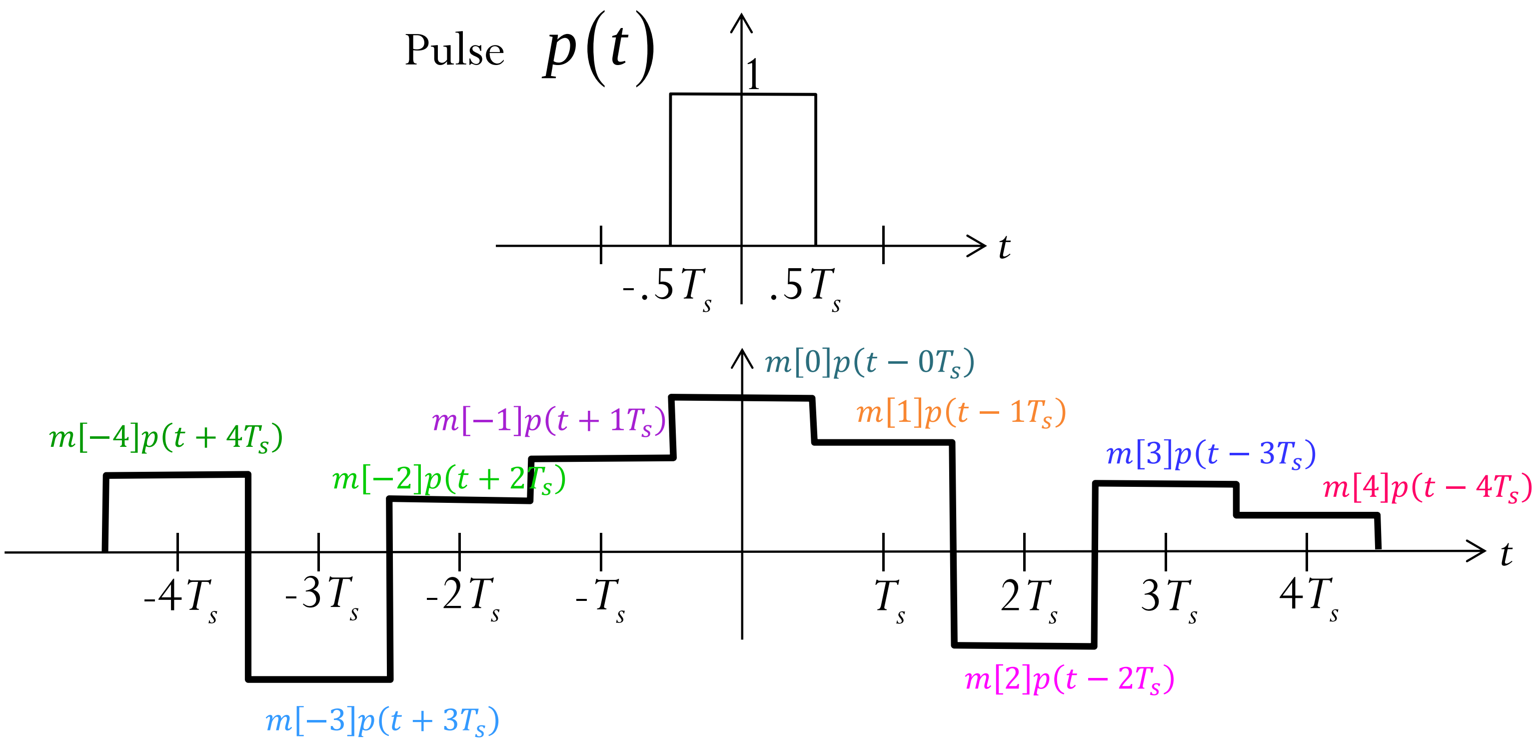
PAM: Example 7.11 (Figure 58)



The height (amplitude) of each pulse is scaled by the corresponding $m[n]$

$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n]\,p(t-nT_s)$$

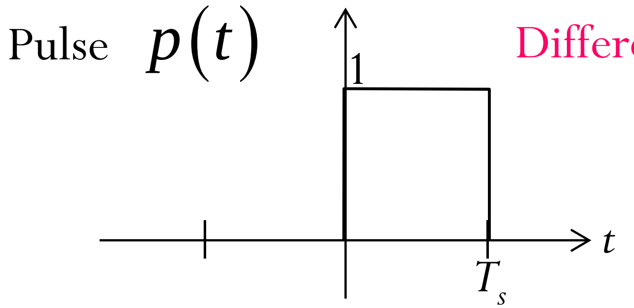
PAM: Example 7.11 (Figure 58)



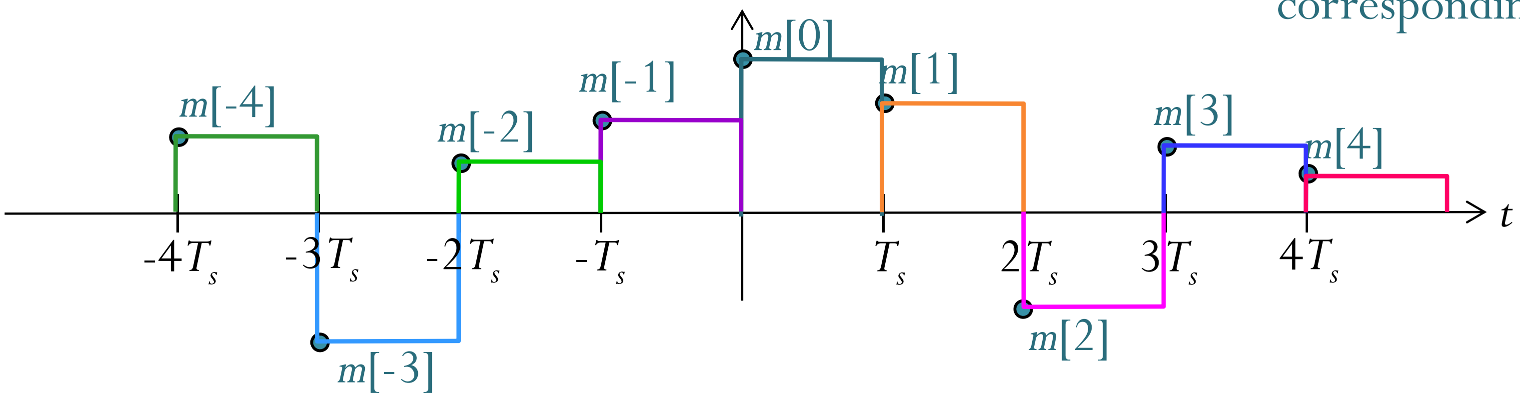
$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n]\, p(t - nT_s)$$

PAM: Example 7.12 (Figure 59)

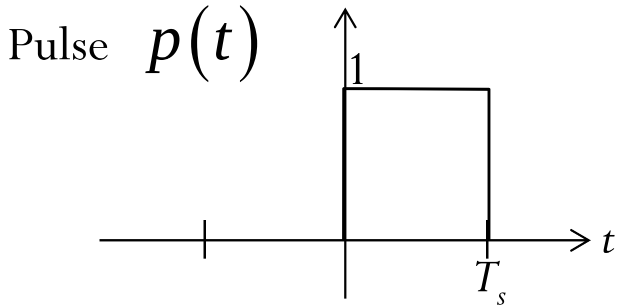
Different pulse is used here.

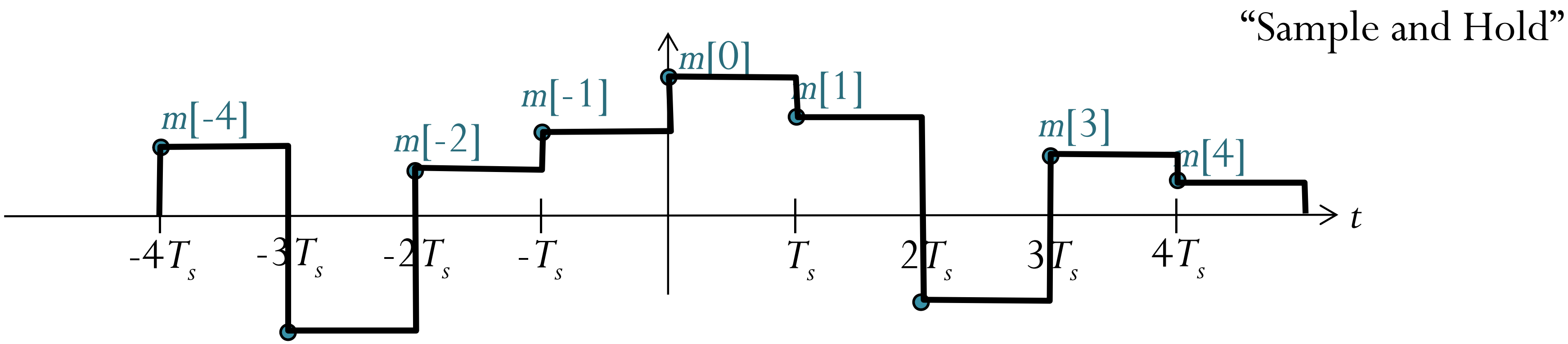


The height (amplitude) of each pulse is scaled by the corresponding $m[n]$



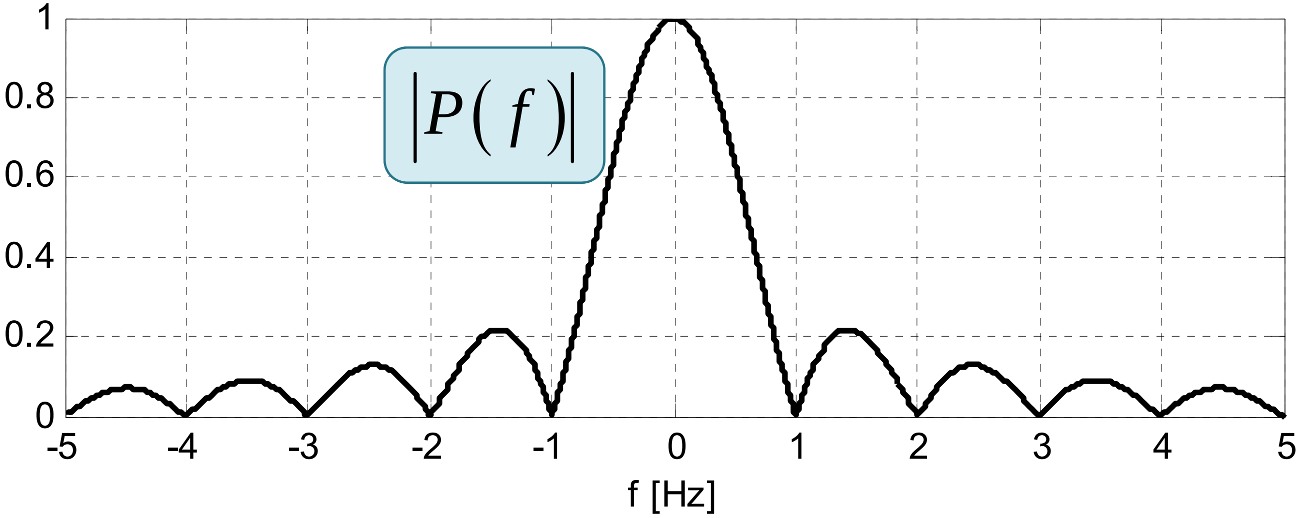
PAM: Example 7.12 (Figure 59)

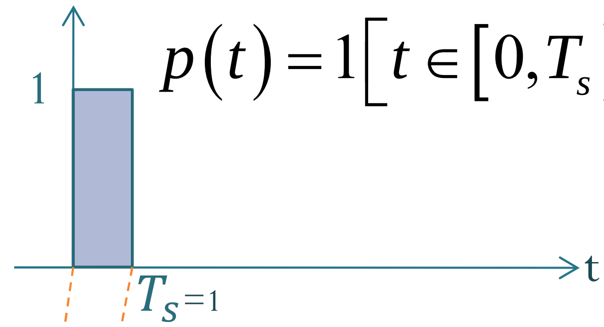




$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n]\, p(t - nT_s)$$

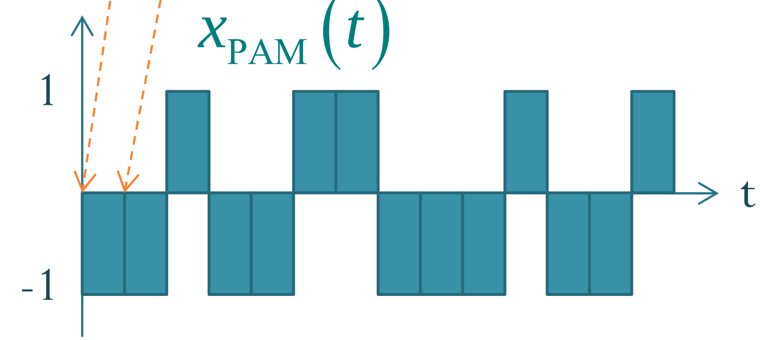
Review: Spectrum of PAM signal





$$p(t) = 1[t \in [0, T_s)]$$

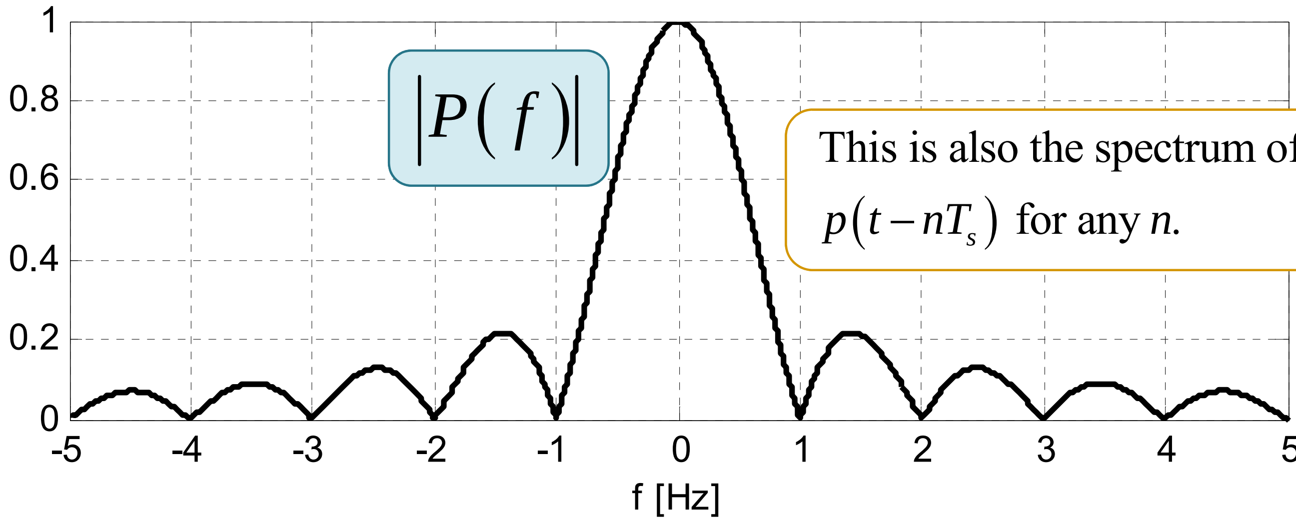
m= [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]



$$x_{\text{PAM}}(t) = \sum_n m[n]\, p(t - nT_s)$$

Can you sketch the spectrum of $x_{\text{PAM}}(t)$?

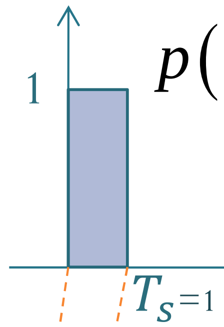
$X_{\text{PAM}}(f)$ (2/4)



$|P(f)|$

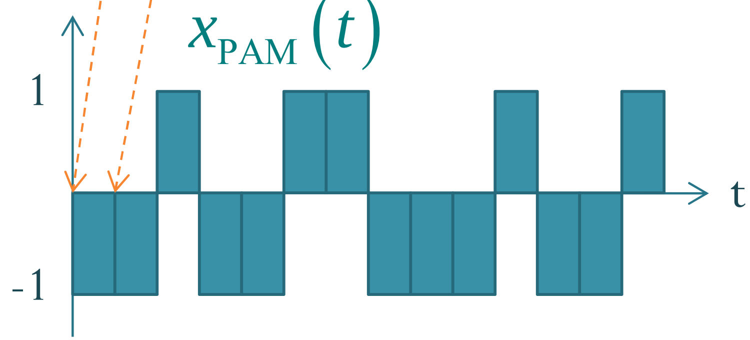
This is also the spectrum of $p(t-nT_s)$ for any n.

$$p(t) = 1\left[t \in [0, T_s)\right]$$



m= [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]

$$x_{\text{PAM}}(t) = \sum_n m[n]\, p(t - nT_s)$$



Does this mean $|X_{\text{PAM}}(f)|$ will simply be a sum of $|P(f)|$ and therefore its shape will be similar to $|P(f)|$?

Important Properties of $\mathcal{F}$

$$\{x*y\}(t) = \int_{-\infty}^{\infty} x(\mu)y(t-\mu)d\mu = \int_{-\infty}^{\infty} x(t-\mu)y(\mu)d\mu$$

Convolution Properties:

$$x * y \overset{\mathcal{F}}{\rightleftharpoons} X \times Y$$

$$x \times y \overset{\mathcal{F}}{\rightleftharpoons} X * Y$$

Note that the magnitude of this is simply $|G(f)|$

Shifting Properties:

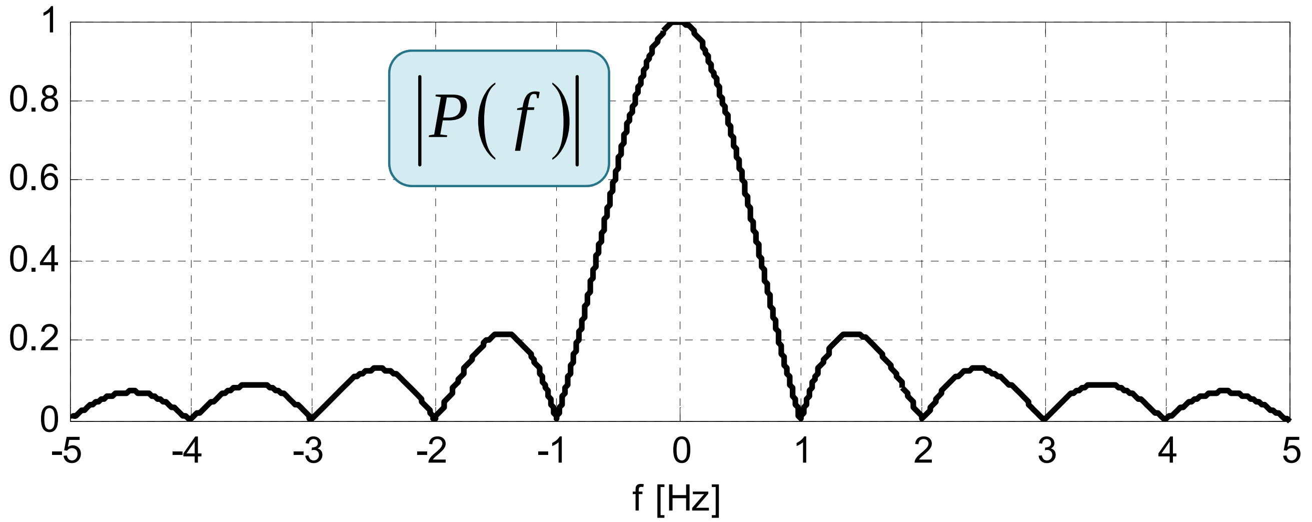
$$g(t-t_0) \overset{\mathcal{F}}{\rightleftharpoons} e^{-j2\pi f t_0} G(f)$$

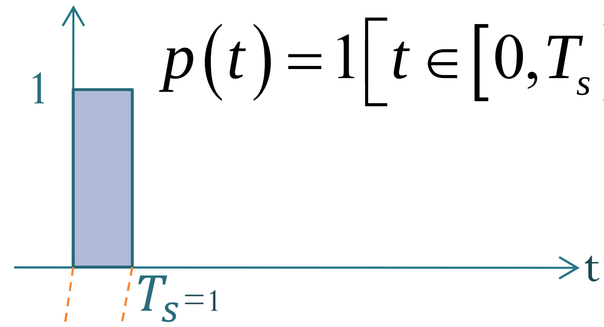
$$e^{j2\pi f_0 t} g(t) \overset{\mathcal{F}}{\rightleftharpoons} G(f-f_0)$$

Modulation:

$$g(t)\cos(2\pi f_c t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{2}G(f-f_c) + \frac{1}{2}G(f+f_c)$$

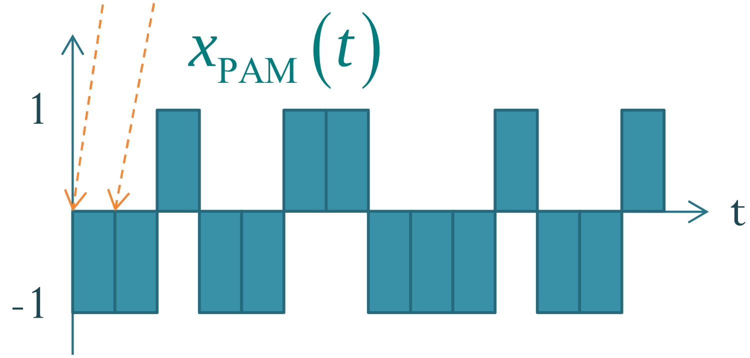
$X_{\text{PAM}}(f)$ (3/4)





$$p(t) = 1\left[t \in [0, T_s)\right]$$

m= [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]



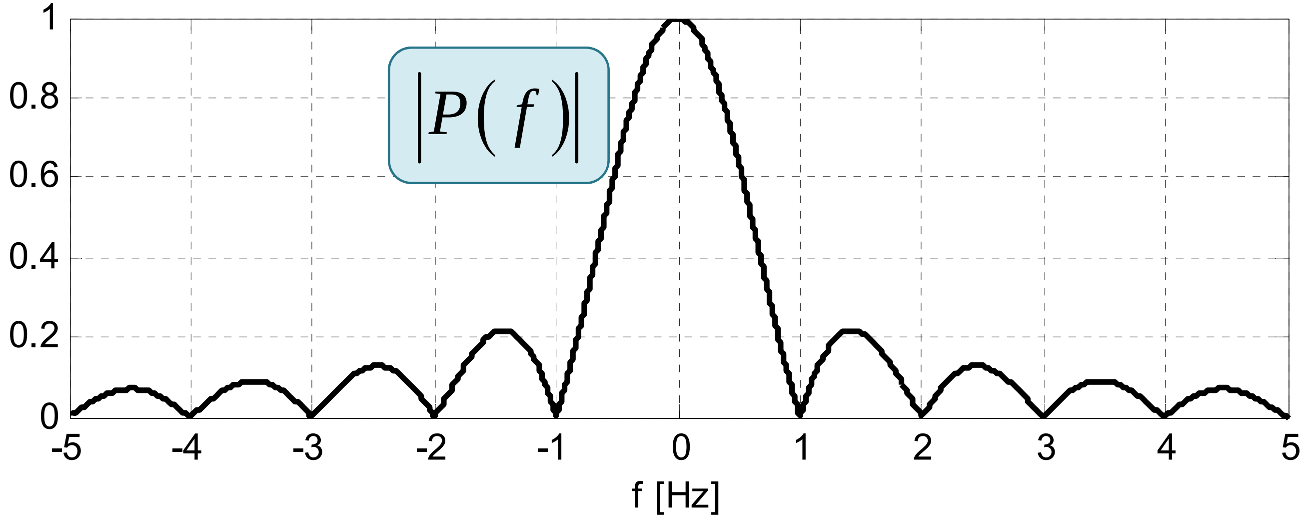
$$x_{\text{PAM}}(t) = \sum_n m[n]\, p(t - nT_s)$$

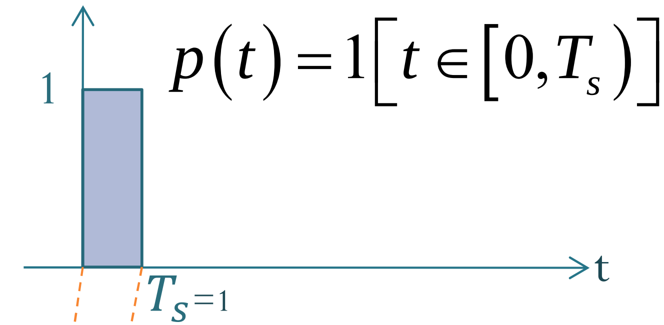
$$\xrightarrow{\mathcal{F}} X_{\text{PAM}}(f) = \sum_n m[n] P(f) e^{-j2\pi f n T_s}$$

$$= P(f) \sum_n m[n] e^{-j2\pi f n T_s}$$

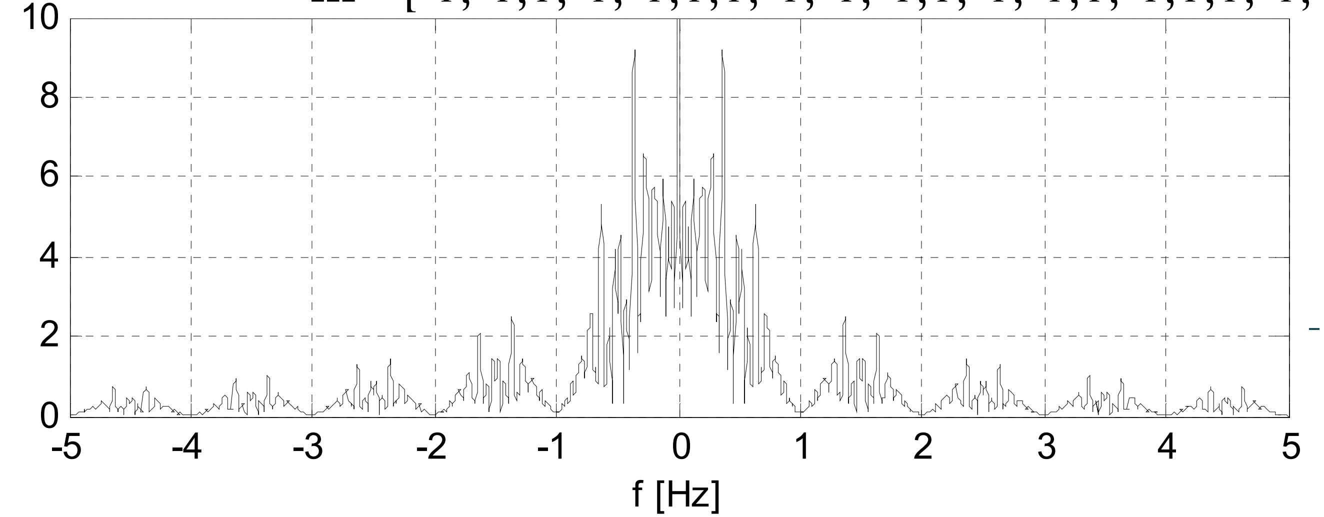
Figure 60

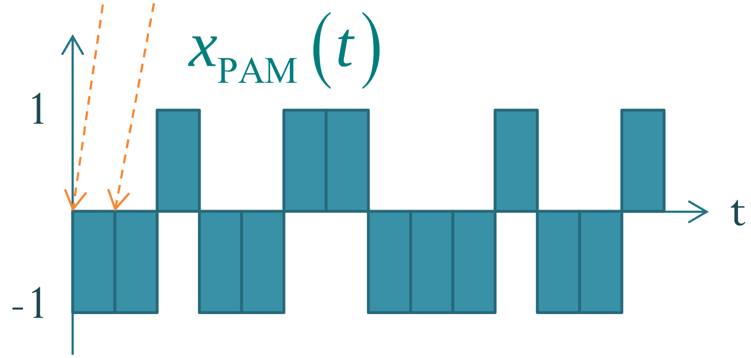
$X_{PAM}(f)$ (4/4)





m= [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]

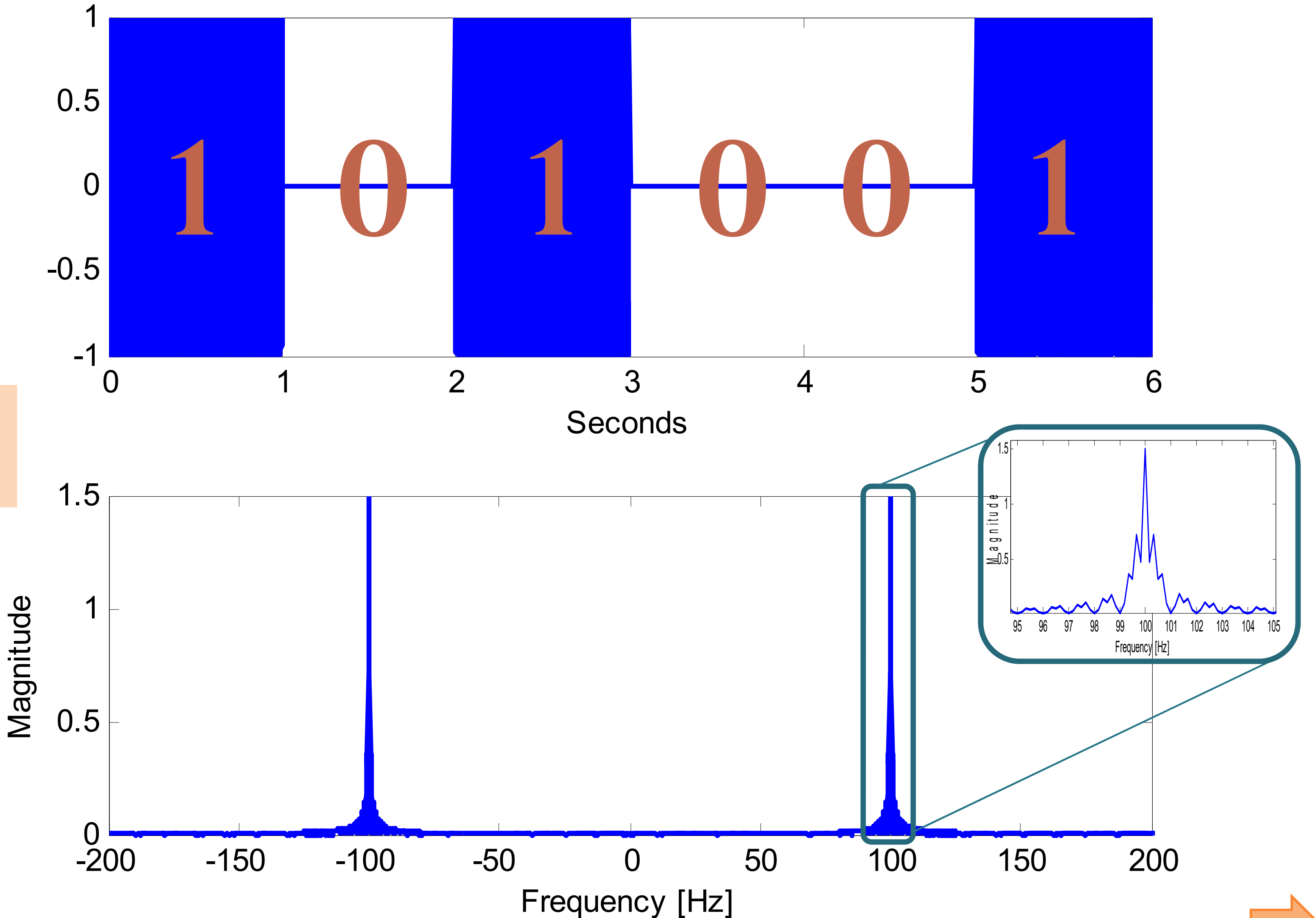




$$x_{\text{PAM}}(t) = \sum_n m[n]\, p(t - nT_s) \xrightarrow{\mathcal{F}} X_{\text{PAM}}(f) = P(f) \sum_n m[n]\, e^{-j2\pi f n T_s}$$

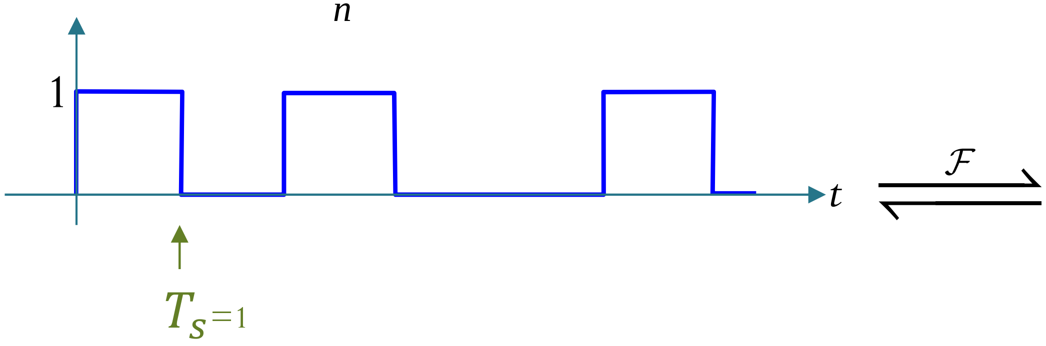
A revisit to an earlier OOK Example

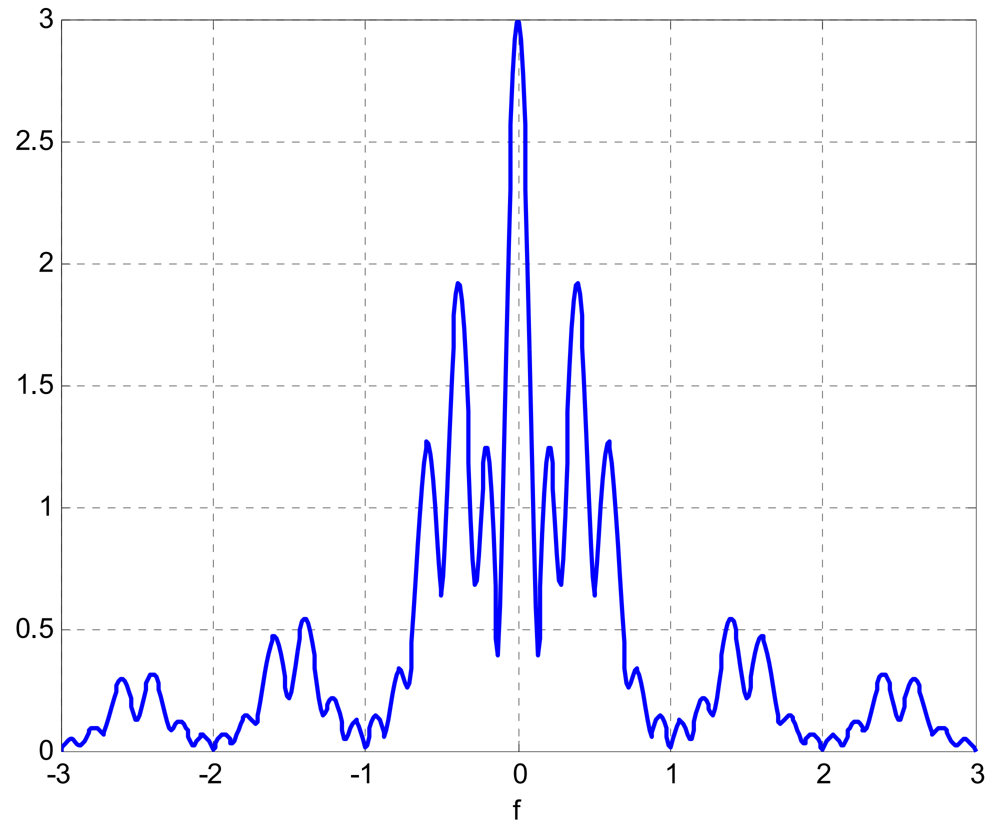
$f_c = 100$ Hz
Bit rate = 1 bps



A revisit to an earlier OOK Example

$$x_{\text{PAM}}(t) = \sum_n m[n]\, p(t - nT_s)$$





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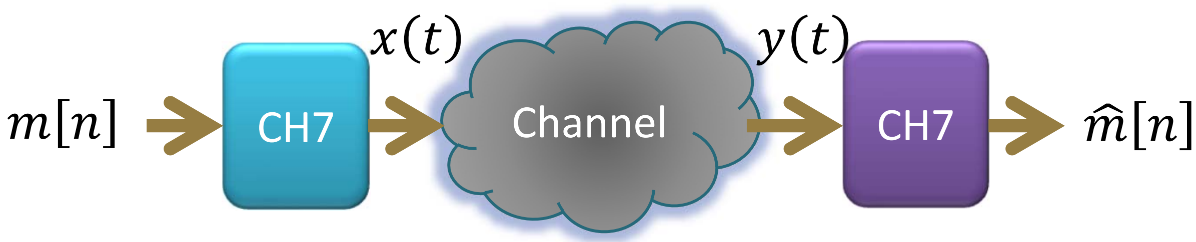
7.3 ISI and Pulse Shaping

Section 7.3

- Start with a discrete-time signal (a sequence of numbers $m[n]$).
- Original Goal: Convert $m[n]$ into an analog signal $x(t)$ that
  - has no inter-symbol interference
  - uses small bandwidth.

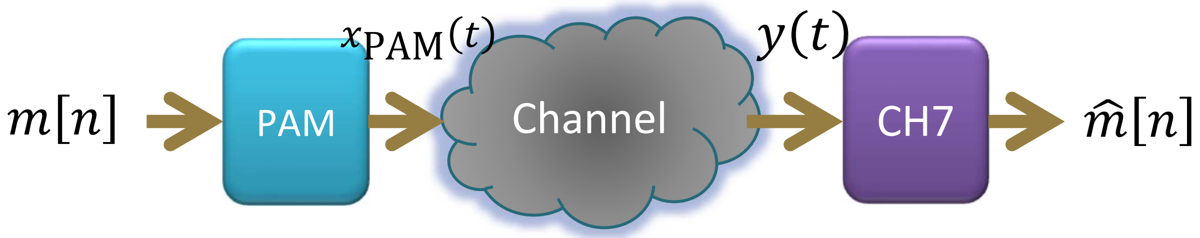


Section 7.3

- Start with a discrete-time signal (a sequence of numbers $m[n]$).
- In Section 7.2, we choose to convert $m[n]$ into an analog signal by using PAM:

$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n]p(t - nT_s)$$

- Current Goal: Choose the pulse shape $p(t)$ so that $x(t)$
  - has no inter-symbol interference
  - uses small bandwidth.



Section 7.3

- Start with a discrete-time signal (a sequence of numbers $m[n]$).
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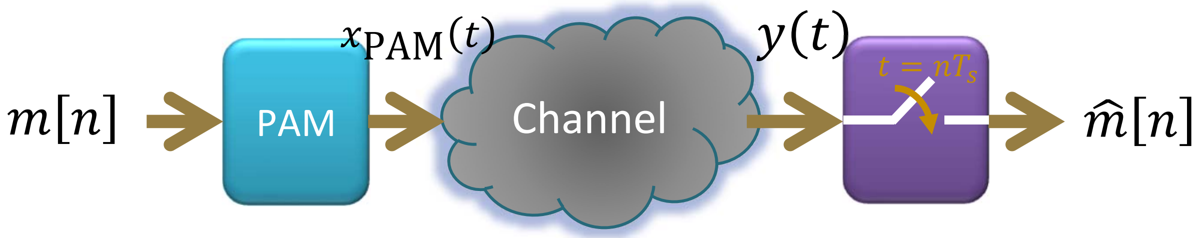
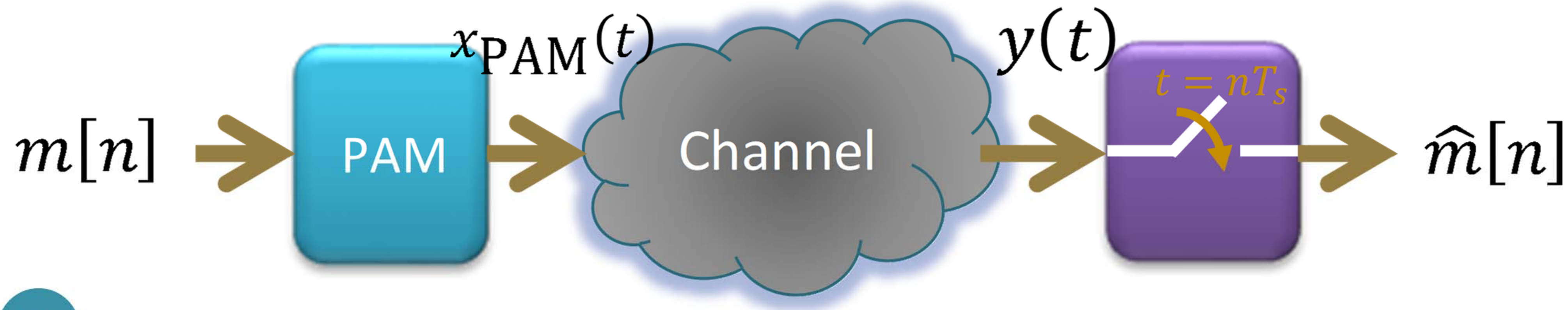


Section 7.3: Conditions for good pulse

- Start with a discrete-time signal (a sequence of numbers $m[n]$).
- In Section 7.2, we choose to convert $m[n]$ into an analog signal by using PAM:

$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n]p(t - nT_s)$$

- Current Goal: Choose the pulse shape $p(t)$ so that $x(t)$
  - has no inter-symbol interference: $\hat{m}(n) = m(n)$ for all n
  - uses small bandwidth.

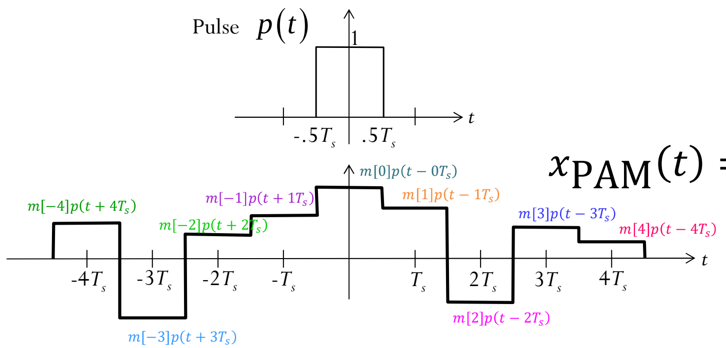


Conditions for good pulse

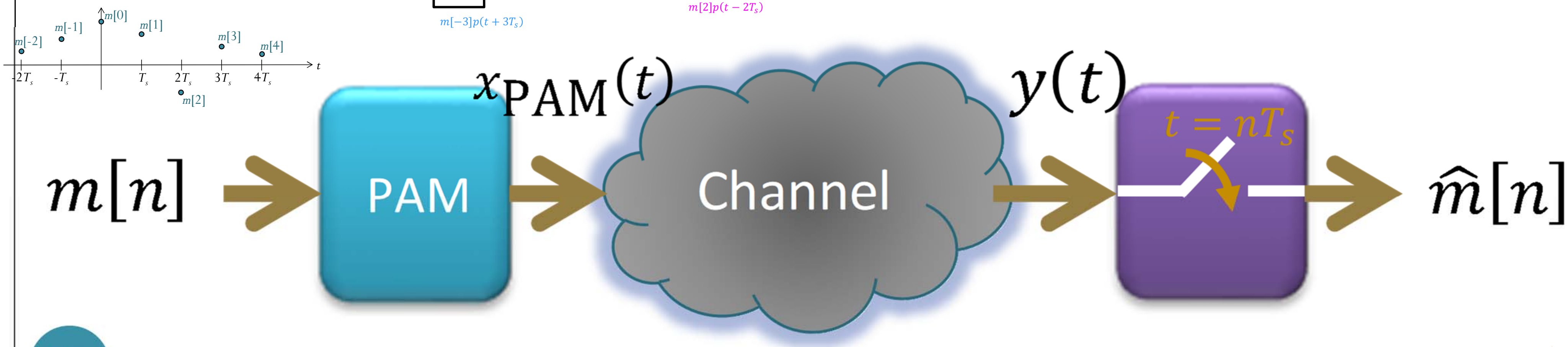
[7.17] Two conditions:

(a1) $\hat{m}(n) = m(n)$ for all n.

(b) $P(f)$ is band-limited (with small BW).



$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n]p(t - nT_s)$$



Conditions for good pulse

[7.17] Two conditions:

(a2) [7.22] $p(t) = \begin{cases} 1, & t = 0, \\ 0, & t = \pm T, \pm 2T, \pm 3T, \ldots \end{cases}$

(b) $P(f)$ is band-limited (with small BW).

These two conditions are in two different domains.

[7.26]

Nyquist's (first) Criterion for Zero ISI

- Two equivalent definitions for **Nyquist pulse**:
- In the **time domain**,

(a2)
$$p(t) = \begin{cases} 1, & t = 0, \\ 0, & t = \pm T, \pm 2T, \pm 3T, \ldots \end{cases}$$

Symbol "duration" "interval"

$\frac{1}{T}$ = **signaling rate** measured in [symbols per second] or [baud]

- In the **frequency domain**,

(a3)
$$\sum_{k=-\infty}^{\infty} P\left(f - \frac{k}{T}\right) \equiv T, \qquad |f| \le \frac{1}{2T}$$

The sum on the left is always periodic with period $\frac{1}{T}$. So, it is sufficient to check the criterion only on this interval.

Conditions for good pulse

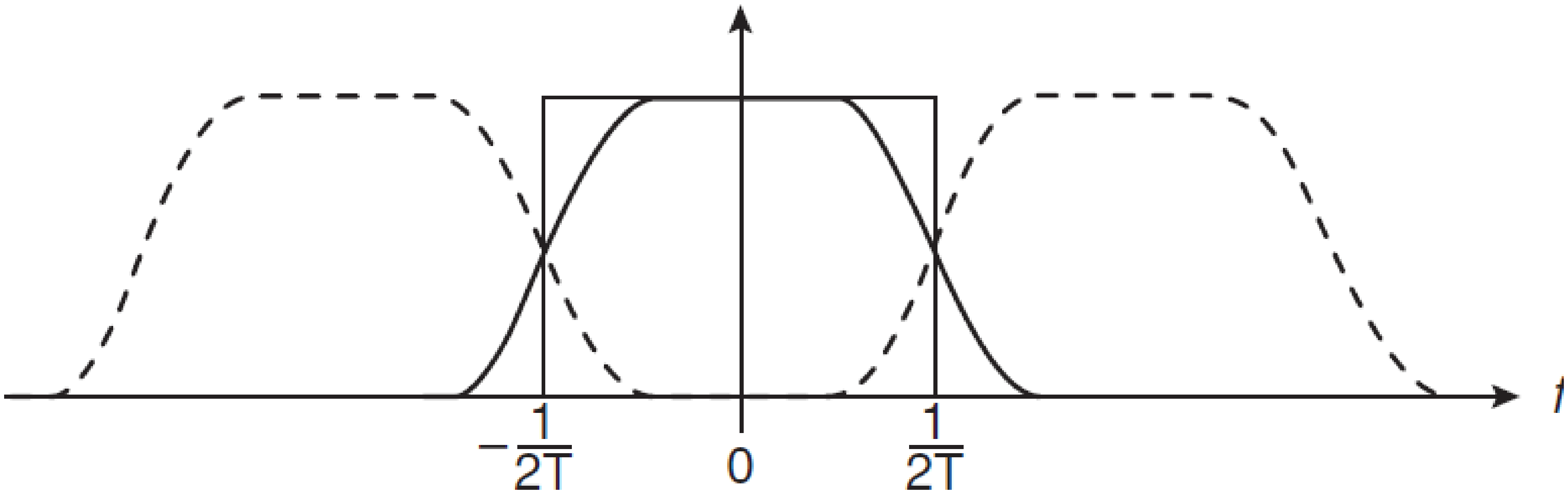
[7.17] Two conditions:

(a3) [7.26] $\sum_{k=-\infty}^{\infty} P\left(f-\frac{k}{T}\right) \equiv T, \quad |f| \leq \frac{1}{2T}$

(b) $P(f)$ is band-limited (with small BW).

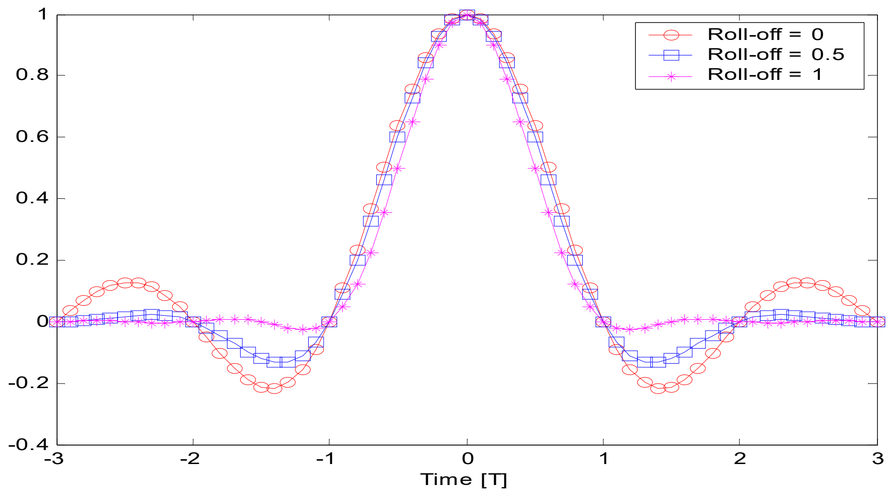
The two conditions are in the same domain.

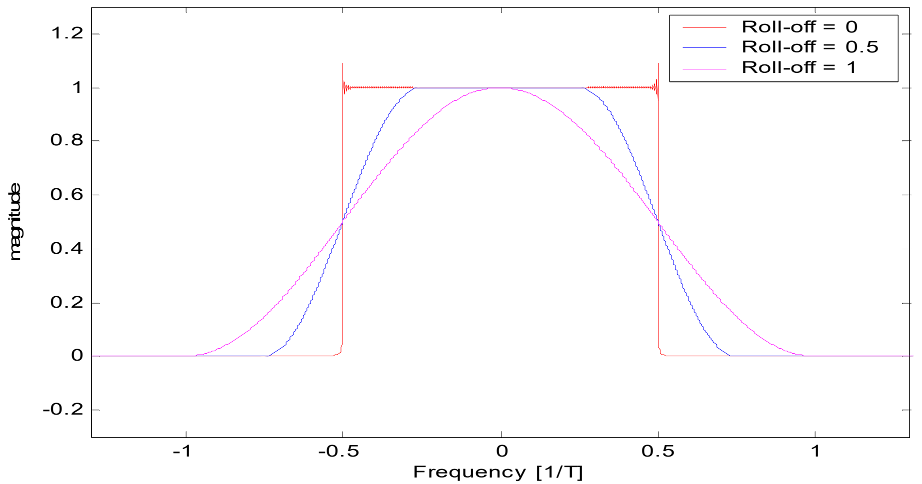
Nyquist criterion



[Blahut, 2008, Fig 2.9]

Raised Cosine Pulses





For fixed nonzero α, the tails decay as $1/t^3$ for large $|t|$.

Although the pulse tails persist for an infinite time, they are eventually small enough so they can be truncated with only negligible perturbations of the zero crossings.

$$p_{\mathrm{RC}}(t;\alpha) = \frac{\cos\frac{\alpha\pi t}{T}}{1-\frac{4\alpha^2 t^2}{T^2}} \operatorname{sinc}\frac{\pi t}{T}$$

$$= \frac{\cos\frac{\alpha\pi t}{T}}{1-\frac{4\alpha^2 t^2}{T^2}} \frac{\sin\frac{\pi t}{T}}{\frac{\pi t}{T}}$$